Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT2 **Branch: B. Tech (All)**

Semester : 2 **Date : 20/04/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The infinite series $1+r+r^2+.....+r^{n-1}$ is convergent if
(A) $|r|<1$ (B) $|r|>1$ (C) $r=1$ (D) $r<-1$

b) The sum of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....$ is
(A) $\log 2$ (B) zero (C) infinite (D) none of these

c) If $f_n=\int_0^{\frac{\pi}{4}}\tan^n x\, dx$, then (f_n+f_{n-2}) is equal to ?
(A) $\frac{1}{n}$ (B) $\frac{1}{n-1}$ (C) $\frac{n}{n-1}$ (D) $\frac{n-1}{n}$

d) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^7 x\, dx$ is
(A) $\frac{32\pi}{35}$ (B) $\frac{32}{35}$ (C) zero (D) $\frac{16}{35}$

e) $\left\lceil\frac{1}{2}\right.\left\lceil\frac{3}{2}\right.\left\lceil\frac{5}{2}\right.=$ _____
(A) $\frac{3}{8}(\pi)^{\frac{3}{2}}$ (B) $\frac{3}{8}(\pi)^{\frac{5}{2}}$ (C) $\frac{3}{8}(\pi)^{\frac{1}{2}}$ (D) $\frac{1}{8}(\pi)^{\frac{3}{2}}$

f) Duplication formula: $\lceil n \left\lceil n+\frac{1}{2}\right.=$ _____
(A) $\frac{\sqrt{\pi}\,\lceil n}{2^{2n-1}}$ (B) $\frac{\sqrt{\pi}\,\lceil 2n}{2^{n-1}}$ (C) $\frac{\sqrt{\pi}\,\lceil 2n}{2^{2n-1}}$ (D) $\frac{\sqrt{\pi}\,\lceil n}{2^{n-1}}$

g) $erf(x)+erf_c(x)$ is equal to
(A) 0 (B) 1 (C) -1 (D) 2

h) $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-2\sin^2\theta}}$ is equal to

(A) $\frac{1}{\sqrt{2}}E\left(\frac{1}{\sqrt{2}}\right)$ (B) $\frac{1}{2}K\left(\frac{1}{\sqrt{2}}\right)$ (C) $\frac{1}{\sqrt{2}}K\left(\frac{1}{\sqrt{2}}\right)$ (D) $\frac{1}{2}E\left(\frac{1}{\sqrt{2}}\right)$

i) The tangents at the origin are obtained by equating to zero
(A) the lowest degree terms (B) the highest degree terms
(C) constant term (D) none of these

j) If the powers of x are even, then the curve is symmetrical about
(A) X – axis (B) Y – axis (C) about both X and Y axes (D) None of these

k) $\int_0^{\frac{\pi}{2}}\int_0^{\infty} e^{-r^2} \cdot r\, dr\, d\theta$ is equal to

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

l) The transformations $x+y=u,\ x-y=v$ transform the area element $dy\ dx$ into $|J|du\ dv$, where $|J|$ is equal to

(A) $\frac{1}{2}$ (B) 1 (C) u (D) none of these

m) The degree and order of the differential equation of all parabolas whose axis is x-axis are
(A) 2, 1 (B) 1, 2 (C) 3, 2 (D) none of these

n) Solution of differential equation $\mathrm{xdy}-\mathrm{ydx}=0$ represents
(A) Rectangular hyperbola (B) Circle whose centre is at origin
(C) Parabola whose vertex is at origin
(D) Straight line passing through origin

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Using reduction formula prove that $\int_0^a x^5\left(2a^2-x^2\right)^{-3}dx=\frac{1}{2}\left(\log 2-\frac{1}{2}\right)$. **(5)**

b) Prove that $\int_0^{\infty}\frac{x^4}{4^x}dx=\frac{24}{(\log 4)^5}$. **(5)**

c) Evaluate: $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}\left(x^2+y^2+z^2\right)dz\ dy\ dx$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Prove that $\int_0^1 x^5\left(1-x^3\right)^{10}dx=\frac{1}{3}B(2,11)$. **(5)**

b) Solve: $\frac{dy}{dx}+2y\tan x=\sin x$ given that $y=0$ when $x=\frac{\pi}{3}$ **(5)**

c) Test the convergence of the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^2}$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) By changing into polar co-ordinates, evaluate the integral **(5)**

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)\,dx\,dy \ .$$

b) Examine the series $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+.....+\frac{x^n}{n^2+1}+.....$ for convergence using ratio test. **(5)**

c) Using reduction formula evaluate: $\int_0^{\infty}\frac{x^4}{(1+x^2)^4}\,dx$ **(4)**

Q-5 **Attempt all questions** **(14)**

a) Solve: $\frac{(x-2y)}{(3x+y)}\frac{dy}{dx}=3x^2-5xy-2y^2$ **(5)**

b) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$ and hence evaluate it. **(5)**

c) Prove that $\int_0^{\infty}\frac{x^4(1+x^5)}{(1+x)^{15}}\,dx=\frac{1}{5005}$. **(4)**

Q-6 **Attempt all questions** **(14)**

a) Examine the series $\sum_{n=1}^{\infty}\frac{x^n}{n^p}$ for convergence using root test. **(5)**

b) Using reduction formula prove that $\int_0^{\pi} x\cos^6 x\,dx=\frac{5\pi^2}{32}$. **(5)**

c) Solve: $(x^2+y^2+1)dx-2xy\,dy=0$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Trace the curve $y^2(2a-x)=x^3$. **(5)**

b) Find the area enclosed by the cardioid $r=a(1-\cos\theta)$. **(5)**

c) Evaluate: $\int_0^{\frac{\pi}{2}}\frac{dx}{\sqrt{\cos x}}$ **(4)**

Q-8 **Attempt all questions** **(14)**

a) For small values of x, show that $\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\left(x-\frac{x^3}{1!3}+\frac{x^5}{2!5}-\frac{x^7}{3!7}+.....\right)$. **(5)**

b) Trace the curve $r=a(1+\cos\theta)$. **(5)**

c) Find the length of the arc of the curve $y=\log\sec x$ from $x=0$ to $x=\frac{\pi}{3}$ **(4)**